Wave packet dynamics with Bose-Einstein condensates

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We study wave packet dynamics of a Bose condensate in a periodically shaken trap. Dichotomy, that is, dynamic splitting of the condensate, and dynamic stabilization are analyzed in analogy with similar phenomena in the domain of atoms in strong laser fields.

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Recently it has become possible to prepare Bose-Einstein condensates of alkali gases [1] with up to 10^7 magnetically trapped atoms. The condensed state is a macroscopically populated quantum state well localized in the magnetic trap. It is, therefore, an ideal tool to study wave packet dynamics under experimentally feasible conditions. There are many interesting quantum phenomena resulting from the electronic wave packet dynamics; in this Letter we refer, in particular, to the phenomena of *wave packet dichotomy and stabilization* exhibited by an electron bound by an atomic potential in presence of a strong laser field [2]. We argue that the same phenomena occur in the dynamics of the condensate wave function. The analogy is based on the fact that in the frame of reference moving with the free electron oscillating in the field, the Kramers-Henneberger frame, the effect of the laser is equivalent to a periodic shaking of the atomic potential along the laser polarization axis. A condensate in a periodically shaken trap could, therefore, *a priori* show a similar behavior.

As the intense laser field drives the electron, the process of ionization of the atom occurs. By increasing the laser intensity, one normally increases the ionization rate. However, for very intense fields of high frequency, this rate eventually starts to decrease with intensity – this is called *atomic stabilization* [2]. In this process the electronic wave packet remains bounded, i.e. well localized in space (without spreading), although highly distorted due to the combined effects of the laser field and the atomic potential . This *effective* atom-laser potential exhibits a double well structure which splits the electronic wave packet into two spatially separate parts; this effect is called *dichotomy*. To achieve stabilization it is necessary to turn on the laser adiabatically in order to ensure that the atomic ground state will evolve to the ground

state of this *effective* atom-laser potential. This type of stabilization [3] has never been observed experimentally [4], since it requires very intense high frequency fields, which currently can only be generated in a form of a very short pulse; as an electron in an atom is highly unstable, it would thus be most likely ionized during the turn-on of such a pulse.

Let us analyze the analogy between the electron and the condensate in more detail. The electron bound by an atomic potential $U(\vec{r})$ and interacting with a laser field of amplitude $\mathcal{E}\vec{e}_z$ (polarized along the z-direction) is, for our purposes, best described in the Kramers-Henneberger frame of reference, in which the interaction with the laser field results in an effective time dependent "atomic" potential:

$$\left[-i\hbar\partial_t - \frac{\hbar^2\vec{\nabla}^2}{2m_e} + U_e(\vec{r} + \alpha_L\sin(\omega_L t)\vec{e}_z) \right]\Psi_e(\vec{r},t) = 0, \quad (1)$$

where $\alpha_L = (e\mathcal{E})/(m_e\omega_L^2)$ is the electron excursion amplitude, while ω_L is the laser frequency.

Consider now a condensate with N atoms in a magnetic trap $V(\vec{r})$ which is periodically shaken along the z-axis. In a Hartree-Fock treatment, the state of the condensate is described by the Gross-Pitaevskii equation (GPE) [5]. It accurately describes the wave function of the condensate Ψ in presence of particle interactions in thermal equilibrium at temperatures well below the critical temperature. Furthermore, the time dependent GPE describes the dynamics of the condensate in more general time dependent conditions [5–7]. We, therefore, have:

$$\left[-i\hbar\partial_t - \frac{\hbar^2\vec{\nabla}^2}{2m} + V(\vec{r} + \alpha(t)\vec{e}_z) + gN|\Psi(\vec{r},t)|^2 \right]\Psi(\vec{r},t) = 0.$$

$$(2)$$

Here $\alpha(t) = \alpha_0\sin(\omega t)$ is the shaking amplitude. If we identify the amplitude α_0 and the frequency ω of the shaking with the electron excursion amplitude α_L and the laser frequency ω_L respectively, Eq.(2) is very similar to Eq.(1). An important difference is the presence of a nonlinear coupling term with a coupling constant $g = 4\pi\hbar^2 a_s/m$, where $a_s > 0$ is the s-wave scattering length. The effects due to the presence of the nonlinear term

as well as the larger mass of the atom will be discussed below. We model the trapping potential by

$$V(\vec{r}) = m(\Omega_x^2 x^2 + \Omega_y^2 y^2 + \Omega_z^2 z^2)/2, \qquad (3)$$

for $V(\vec{r}) < V_c$, and $V(\vec{r}) = V_c$ otherwise. This is a harmonic potential with frequencies $\Omega_{x,y,z}$ which is cut at an energy V_c. It thus resembles $U_e(\vec{r})$, in particular we have that $V(\vec{r} \to \infty)$ like $U_e(\vec{r} \to \infty)$ is constant. This trapping potential can be realized as an effective (adiabatic) trap potential "dressed" by the microwave coupling between a trapped and an untrapped state. We will discuss this model later on in the paper. Another possible realization would be a condensate in a dipole trap potential formed by a strong off-resonant laser field. The condensation might take place in such a trap, or the magnetically trapped condensate may be loaded into it. Note that in absence of the cut V_c, i.e. for an exactly harmonic potential the shaking of the trap would lead to an undistorted oscillation of the condensate wave function.

If the time scale $1/\omega_L$ in Eq.(1) is shorter than the typical time scale of the electronic motion, the combined laser-atom potential can be replaced by its time average over one laser period; ideally the wave function of the electron will then evolve adiabatically from a state in the bare atomic potential into the corresponding state of this time-averaged atom-laser potential [2]. For a large enough amplitude α_L, this time average potential will have a double well structure and thus the electron wave function will exhibit dichotomy. Using the same reasoning for the condensate we replace the GPE (Eq.(2)) by a GPE with a time-averaged potential if the time scale $\sim 1/\omega$ of the shaking is shorter than all other relevant time scales:

$$\left[-i\hbar\partial_t - \frac{\hbar^2\vec{\nabla}^2}{2m} + V_{\text{eff}}(\vec{r}, \alpha_0) + gN|\Psi(\vec{r}, t)|^2 \right] \Psi(\vec{r}, t) = 0, \qquad (4)$$

where the time-averaged potential is given by

$$V_{\text{eff}}(\vec{r}, \alpha_0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\varphi\, V(\vec{r} + \alpha_0 \sin(\varphi)\mathbf{e}_z); \ \varphi = \omega t. \quad (5)$$

For sufficiently large shaking amplitudes, a double well structure appears as illustrated in Fig. 1(a). We therefore expect dichotomy as well as stabilization to appear also, due to the trapping of the atomic wave function in the two potential wells of V_{eff} near the turning points of the oscillation. Indeed, for large enough α_0, stable states are dynamically formed in the double well. Even though the particles in such states are sometimes out of the trap, no "ionization" occurs. This is due to the fact that the time scale for particle motion is much larger than the time scale $1/\omega$ of shaking, so that the particles do not have enough time to react on being momentarily out of the trap. Due to the large atomic mass, the requirements on ω are in fact much less stringent than in the electronic case.

The initial state of the condensate for temperature $T \simeq 0$ is given by the lowest eigenstate Ψ_0 of the time independent GPE

$$\left[-\mu - \frac{\hbar^2\vec{\nabla}^2}{2m} + V(\vec{r}) + Ng|\Psi_0(\vec{r})|^2 \right]\Psi_0(\vec{r}) = 0. \qquad (6)$$

where μ is the chemical potential. By replacing V by V_{eff} in Eq.(6) we determine the ground state of the system in presence of the double well potential. The corresponding chemical potential is μ_{eff}.
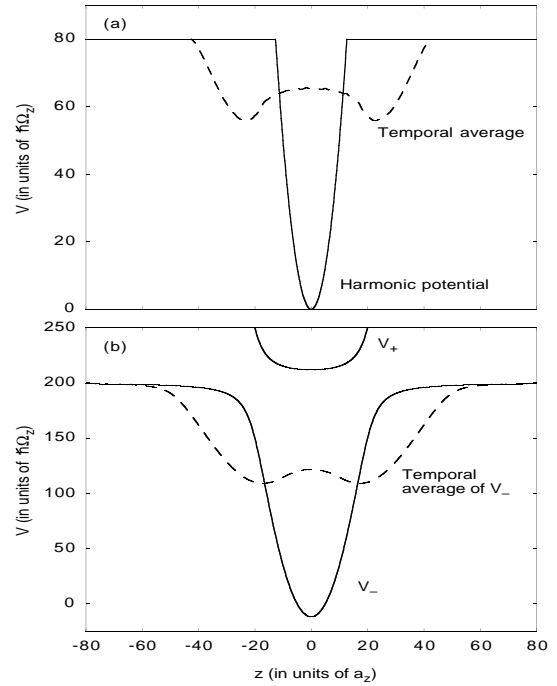

FIG. 1. (a) Cut harmonic potential(solid line) and time averaged potential (dashed line) for $\alpha_0 = 30a_z$ and $V_c = 80\hbar\Omega_z$; $a_z = (\hbar/2m\Omega_z)^{1/2}$ and $gN = 100$. (b) Dressed state potentials (V_+ and V_-) and time averaged potential of the lower one (V_-) from Eq.(9); $\alpha_0 = 30a_z$, $\omega_R = 100\Omega_z$, $\Delta = 200\Omega_z$.

In contrast to Eq.(1), the GPE accounts for atomic interactions, so that the condensate splitting depends on gN. With increasing N the wave function will at some point overcome the potential barrier between the wells in V_{eff}, and the splitting will disappear. This will happen when the effective chemical potential (μ_{eff}) in the double well potential V_{eff} exceeds the height of the double well. Therefore the number N is limited from above.

The validity of the above picture depends on the turn-on time of the shaking. Ideally, an adiabatic switching will transfer the ground state of the GPE (Eq.(6)) at $t = 0$ to the ground state of the GPE with V replaced V_{eff}. Too rapid passage from the cut-harmonic potential into the double well form induces transitions to the higher states of V_{eff} and even into the continuum. To avoid these transitions, we turn on the shaking gradually, setting

$$\alpha(t) = \begin{cases} \alpha_0 \sin^2\left(\frac{\pi}{2}\frac{t}{t_{\text{on}}}\right)\sin(\omega t), & \text{for } 0 \leq t \leq t_{\text{on}} \\ \\ \alpha_0 \sin(\omega t), & \text{for } t \geq t_{\text{on}} \end{cases} \tag{7}$$

The turn-on time t_{on} must be $\gg 2\pi/\omega$; in practice we take $t_{\text{on}} \geq (50-150) \times 2\pi/\omega \simeq 2\pi\alpha_0/a_z\omega$, where $a_z = \sqrt{\hbar/2m\Omega_z}$.

A typical experimental sequence to demonstrate the above phenomena will consist in the following. Initially, the system is prepared in the state Ψ_0 of Eq.(6). At time $t = 0$ we slowly start shaking the trap along the z-axis (Eq.(7)). Let us first assume that the perpendicular motion does not play a significant role, and consider the motion along the z-axis only. In Fig. 2(a) we show the the time evolution of the 1D-condensate wave function. The parameters used are the same as for Fig. 1(a), i.e. $\omega = 10\Omega_z$, $V_c = 80\hbar\Omega_z$, $\alpha_0 = 30a_z$, and $gN = 100$. The gradual splitting of the condensate wave function can be clearly seen. Moreover almost all particles remain trapped.

Due to the cut V_c in the trapping potential (Eq.(3)) atoms may escape from the trap. However, as can bee seen from Fig. 2(a), almost no atoms do it; this indicates stabilization. To study this phenomenon in more detail we lower the cut-off energy V_c to $50\hbar\Omega_z$ to favor the escape from the trap. The escape rate is calculated using standard absorbing boundary conditions [2](b); we observe a *decrease* in the escape rate if the shaking amplitude α_0, *increases*. More specifically, increasing α_0 from $15a_z$ to $20a_z$, the escape rate decreases by a factor $\simeq 2$. Due to the large atomic mass the escape rate is overall very small, i.e. $\leq 1\%$ of the trapped population per 100 shaking cycles. For the same reason, the condensate stabilization occurs already for relatively small shaking frequencies $\hbar\omega < V_c - \mu_{\text{eff}}$. The condensate stabilization is, therefore, more pronounced and can be achieved in experimentally more accessible conditions than its analog in strong field dynamics of electrons.
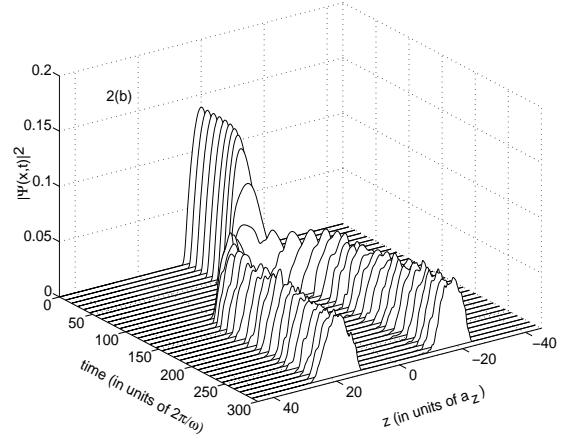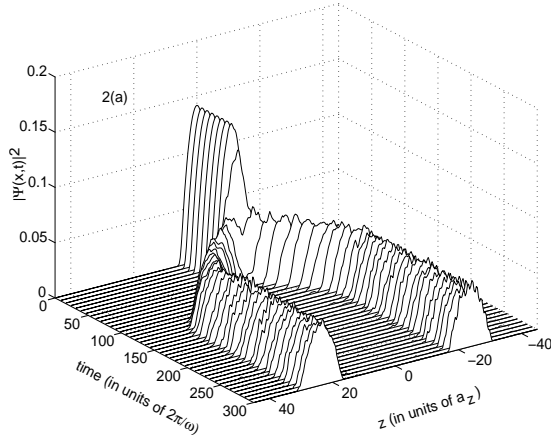

FIG. 2. (a) Time evolution of the condensate density $|\Psi(z,t)|^2$ undergoing 300 shaking cycles; $\alpha_0 = 30a_z$, $\omega = 10\Omega_z$, $t_{\text{on}} = 150 \times (2\pi/\omega)$, $V_c = 80\hbar\Omega_z$. The nonlinear coupling is $gN = 100$, that corresponds to $\hbar\mu = 14.13\Omega_z$; (b) Same as (a) for the total condensate density $|\Psi(z,t)|^2 = |\Psi_1(z,t)|^2 + |\Psi_0(z,t)|^2$, calculated from the 1D two state model; $\alpha_0 = 30a_z$, $\omega = 2.5\Omega_z$, $t_{\text{on}} = 150 \times (2\pi/\omega)$, $\omega_R = 100\Omega_z$, $\Delta = 200\Omega_z$.

We turn now to a discussion of a model with two internal states, a trapped state $(F, m_F \neq 0)$ and an untrapped state $(F', m_F = 0)$. They are coupled via a microwave field which allows coherent transitions between the states [8]. In the rotating wave approximation the GPE is given by

$$\left[-i\hbar\partial_t - \frac{\hbar^2\nabla^2}{2m} + gN(|\Psi_0(\mathbf{r},t)|^2 + |\Psi_1(\mathbf{r},t)|^2)\right.$$
$$\left. + \begin{pmatrix} m\Omega_z^2(z+\alpha(t))^2/2 & \hbar\omega_R/2 \\ \hbar\omega_R/2 & \hbar\Delta \end{pmatrix}\right]\begin{pmatrix} \Psi_0(\mathbf{r},t) \\ \Psi_1(\mathbf{r},t) \end{pmatrix} = 0. \tag{8}$$

Here Ψ_0, Ψ_1 are the wave functions of atoms in the trapped and untrapped state normalized to the respective fraction of atoms in these states, Δ is the detuning of the microwave from the transition frequency, and ω_R is the Rabi frequency of the microwave transition. For simplicity we assume all coupling constants equal to g. For a large enough Rabi frequency ω_R, the coupled states can be replaced by uncoupled dressed states with energies:

$$V_{\pm}(z,t) = \frac{1}{2}\left[m\Omega_z^2(z+\alpha(t))^2/2 + \hbar\Delta\right.$$
$$\left. \pm \sqrt{\left(m\Omega_z^2(z+\alpha(t))^2/2 - \hbar\Delta\right)^2 + \hbar^2\omega_R^2}\right]. \tag{9}$$

The potential V_- resembles our model potential; this is illustrated in Fig. 1(b) in which we also plot time averages of the time dependent dressed state potential. Obviously Eq.(2), with V replaced by V_-, is not exact, since it neglects entirely the non-adiabatic transitions to an upper branch of the dressed potential. The model is nevertheless reasonable, since the exact numerical treatment based of the two–component GPE leads to very

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similar results, as shown in Fig. 2(b). In the model of Eq.(8) we account rigorously for non-adiabatic (Landau-Zener) transitions from the lower to the upper "dressed" state manifold. Obviously, some of the atoms may cease to oscillate in the lower potential and instead they may be transferred to the upper "dressed" potential, destroying the dichotomy. In order to avoid the Landau-Zener transitions ω has to be smaller than a certain critical value [9]. On the other hand for the validity of the time-averaged potential ω has a lower limit of approximately $2\Omega_z$. In consequence dichotomy and stabilization in a realistic system occur in a limited range of ω's.

Finally, we have generalized our study to a 3D case. We chose parameters that resemble those of the MIT experiment [1], that is a cigar shaped trap with a small Ω_z and equal frequencies perpendicular to it. We assume the shaking occurs along the long z-axis. As shown in Fig. 3 the presence of perpendicular motion does not invalidate the conclusions from the 1D approximation: the splitting of the condensate is clearly visible.
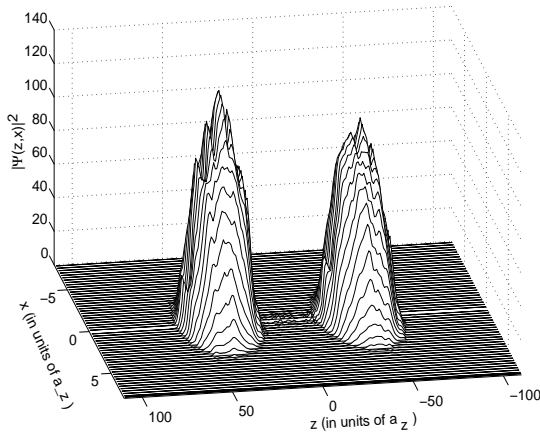

FIG. 3. 2D cross–section of the condensate density $|\Psi(x, z, t)|^2$ for a cigar shaped trap at $t = 400(2\pi/\omega)$; $\alpha_0 = 60a_z$, $\omega = 10\Omega_z$, $t_{on} = 250 \times (2\pi/\omega)$, $V_c = 30\hbar\Omega_z$; $gN = 100$, and $\Omega_x = \Omega_y = 5\Omega_z$.

One should mention that the completeness of our analysis requires to check the stability of the solutions of the time dependent GPE. Unstable solutions might lead to a depletion of the condensate [10]. In a first attempt we verified that the solutions of an oscillating condensate in a harmonic trap lead to stable solutions of the GPE, and therefore to no depletion.

Summarizing, we have shown that Bose-Einstein condensates are ideal tools to study wave packet behavior first predicted in the realm of atoms in superstrong laser fields. We have shown that both, dichotomy and stabilization against atom escape can be achieved in condensates. Although these effects are very difficult to realize in electron-atom systems, their observation in condensates seems quite feasible. Furthermore, we believe that wave packet dynamics of condensates might lead to interesting possibilities of condensate state engineering. For instance, so far double peaked condensates have been created using laser "knives" that cut a single condensate into two parts. We offer here an alternative method to achieve a similar dichotomy in a more controlled way which opens new perspectives for condensate interference studies.

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